The Mathematical Model of Non-Equilibrium Effects in Water-Oil Displacement

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Abstract

Forced oil-water displacement and spontaneous countercurrent imbibition are crucial mechanisms of secondary oil recovery. The classical mathematical models of these phenomena are based on the fundamental assumption that in both these unsteady flows a local phase equilibrium is reached in the vicinity of every point. Thus, the water and oil flows are locally redistributed over their flow paths similarly to steady flows. This assumption allowed the investigators to further assume that the relative phase permeabilities and the capillary pressure are universal functions of the local water saturation, which can be obtained from steady-state flow experiments. The last assumption leads to a mathematical model consisting of a closed system of equations for fluid flow properties (velocity, pressure) and water saturation. This model is currently used as a basis for predictions of water-oil displacement with numerical simulations.

However, at the water front in the water-oil displacement, as well as in capillary imbibition, the characteristic times of both processes are comparable with the times of redistribution of flow paths between oil and water. Therefore, the non-equilibrium effects should be taken into account. We present here a refined and extended mathematical model for the non-equilibrium two-phase (e.g., water-oil) flows. The basic problem formulation as well as the more specific equations are given, and the results of comparison with experiments are presented and discussed.

1. Introduction

The problem of simultaneous flow of immiscible fluids in porous media, and, in particular, the problem of water-oil displacement, both forced and spontaneous, is fundamental to the modern simulations of transport in porous media. This problem is also important for engineering applications, especially in the mathematical simulation of the development of oil deposits.

The classical model of simultaneous flow of immiscible fluids in porous media was constructed in late thirties-early forties by the distinguished American scientists and engineers M. Muskat and M.C. Leverett, and their associates. The model was based on the assumption of the local equilibrium, according to which the relative phase permeabilities and the capillary pressure can be expressed through the universal functions of the local saturation.

The Muskat-Leverett theory was in the past and is nowadays of fundamental importance for the engineering practice of the development of oil deposits. Moreover, this theory leads to new mathematical problems involving specific instructive partial differential equations. It is interesting to note that some of these equations were independently introduced later as simplified model equations of gas dynamics.

Gradually, however, it was recognized that the classical Muskat-Leverett model is not quite adequate, especially for many practically important flows. In particular, it seems to be inadequate for the capillary countercurrent imbibition of a porous block initially filled with oil, one of the basic processes involved in oil recovery, and for the even more important problem of flow near the water-oil displacement front. The usual argument in favor of the local equilibrium is based on the assumption that a representative sampling volume of the water-oil saturated porous medium has the size not too much exceeding the size of the porous channels. In fact, it happens that it is not always the case and the non-equilibrium effects are of importance.

A model, which made it possible to take into account the non-equilibrium effects, was proposed and developed by the first author and his colleagues. This model was gradually corrected and modified. It was confirmed by laboratory and numerical experiments. In its turn, this model leads to non-traditional mathematical problems.

In this paper, the physical model of the non-equilibrium effects in a simultaneous flow of two immiscible fluids in porous media is presented as we see it now. We give here the

basic problem statements and more special equations. We also discuss some peculiar properties of the solutions to the capillary imbibition problem clearly demonstrating arising new effects. This paper is considered as a basis for further investigations.

2. Physical model and basic equations

2.1 The basic properties of the flow of two immiscible fluids in a porous medium: generalized Darcy law and conservation laws. We begin by an assumption, which usually is not explicitly formulated, but actually is one fundamental. This assumption is as follows. Consider two-phase water-oil flow (more generally speaking, wetting and non-wetting immiscible fluids) in an isotropic and homogeneous porous medium. Then, for a given fluid (e.g. oil), the other one (water) and the porous skeleton of the stratum can be considered together as an *effective* porous medium. Physically, it means that for a given fluid, the other fluid creates an additional drag, i.e. the *lubrication* effects for a given fluid do not exist. This assumption makes it possible to apply to the two-phase horizontal flows in an isotropic porous medium a generalized Darcy law in the form:

$$u_w = -\frac{kk_{rw}}{\mu_w}\nabla p_w \qquad\qquad u_o = -\frac{kk_{ro}}{\mu_o}\nabla p_o$$

$$p_o - p_w = \gamma\sqrt{\frac{\phi}{k}}J \qquad\qquad (1)$$

Here the subscripts w and o correspond, respectively, to the water (wetting) and the oil (non-wetting) fluids, u_i are the fluxes of the components, p_i are their pressures, and μ_i are their dynamic viscosities, $i=o,w$. Furthermore, k is the absolute permeability of the porous medium determined from one-phase flow experiments; ϕ is its porosity, i.e., the relative volume occupied by the pores; γ is the surface tension at the water-oil interface. The dimensionless quantities k_{rw} and k_{ro}, which according to our basic assumptions satisfy the inequalities $0 \le k_{ri} \le 1$, are called the *relative permeabilities*. The function J, the dimensionless capillary pressure, has the name of the *Leverett function* in honor of M. C. Leverett.

The mass conservation laws for both components of the mixture have the form:

$$\phi\frac{\partial S}{\partial t} + \nabla\cdot u_w = 0 \qquad \phi\frac{\partial(1-S)}{\partial t} + \nabla\cdot u_o = 0 \quad (2)$$

Here S is the *saturation*, the fractional pore volume occupied by water, and t is the time. By taking the sum of equations (2), we obtain an important equation of the fluid incompressibility

$$\nabla\cdot u = 0 \qquad\qquad (3)$$

Here $u=u_w+u_o$ is the bulk flux.

2.2 The classical Muskat-Meres-Leverett mathematical model of two-phase flow in porous media. The system (1)-(2) is not closed until the functions k_{rw}, k_{ro} and J are properly determined. The classical two-phase flow model proposed by M. Muskat, M. Meres [1] and M.C. Leverett [2] (see also a more recent book [3]) is based upon the fundamental assumption that the local state of the flow is *universal* and *fully equilibrium*. This means that the functions k_{rw}, k_{ro} and J are functions of the actual water saturation S, identical for all processes involving two given fluid components and the rock:

$$k_{rw} = k_{rw}(S), k_{ro} = k_{ro}(S), \qquad J = J(S) \qquad (4)$$

Thus, if these functions are known, the system (1)-(3) **is** closed. This mathematical model found numerous applications and nowadays it forms the basis of numerical simulations of the development of oil deposits throughout the world.

2.3 Non-equilibrium effects. As the functions k_{rw}, k_{ro} and J are, according to the classical model, universal, in principle, they can be obtained from any two-phase flow experiment. In particular, they can be obtained from experiments with steady flows of mixtures at constant water saturation S through a cylindrical core. Such experiments, indeed, were performed, and a generally accepted characteristic structure of these functions is presented in **Fig. 1**.
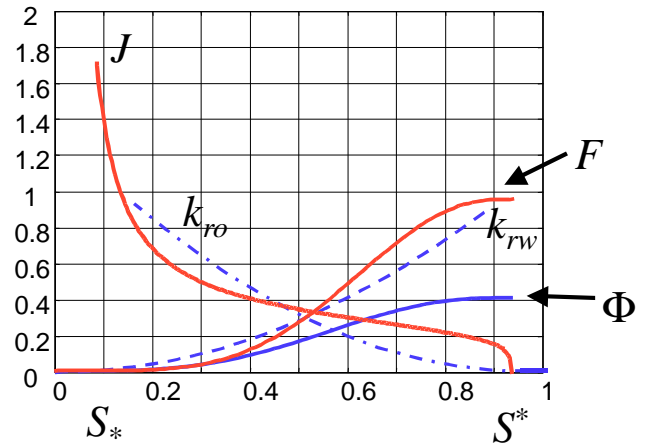

Fig. 1. The qualitative form of the universal functions.

Let us formulate rigorously the properties of these functions, which will be used below. The function k_{rw} is a monotone non-decreasing smooth function. It is equal to zero for $0 \le S \le S_*$, where S_* is a certain constant $0 \le S_* < 1$. In other words, if the water saturation becomes lower or equal to S_*, then the water flow stops. The function $k_{rw}(S)$, in accordance with the experimental data, will be assumed further to have multiple (at least two) zero derivatives at $S=S_*$. The function $k_{ro}(S)$ is monotone non-increasing. It is equal to one for $0 \le S \le S_*$, and equal to zero at $S^* \le S \le 1$, where S^* is again a constant such that $S_* < S^* \le 1$ and the oil flow stops as the water saturation exceeds S^*. The function $k_{ro}(S)$ is also assumed to have multiple zero derivatives at $S=S^*$. Finally, the

Leverett function $J(S)$ is monotonically decreasing. It is equal to infinity at $S=S_*$, equal to zero at $S=S^*$ and the derivative $J'(S)$ at $S=S^*$ is equal to $-\infty$. For simplicity, it was assumed in Fig. 1 that $S_*=0$ and $S^*=1$. We will use this assumption everywhere below.

As it has been mentioned before, the classical model outlined in Section 2.2 played a decisive role in the numerical simulations of the development of oil deposits. However, for strongly unsteady flows, like the capillary imbibition of a porous block, especially at its first stage, or the flow near the water-oil displacement front, which are of crucial importance for the calculations of the oil recovery, the classical model seems to be not quite adequate. Indeed, it is well known to the experimentalists that before reaching the steady state needed to measure the relative phase permeabilities and the Leverett function it is necessary to pump through a porous specimen 5-6 pore volumes of the mixture. The physical reason for the prolonged time required to change the fluid saturations from one steady-state value to another is transparent. In a steady-state flow, there exist separate networks of flow paths connecting the flowing portion of each phase. The flow paths of the nonwetting fluid, oil, are within a *fixed* network of pores and the fluid flows through the central parts of these pores. The wetting fluid, water, remains connected throughout most, if not all, pores in the porous medium. Water occupies the smallest pores, which have capillary entry pressures above the current capillary pressure level. In the pores where both fluids flow, the water flows along the pore corners in filaments and in the pore-wall roughness. *Any* change of the capillary pressure results in the rearrangement of both flow networks resulting in changes of (1) their connectivity, and (2) their overall hydraulic conductances for the fluids flowing in them. A detailed investigation of the microscopic, pore-level mechanisms governing such a rearrangement was performed in [4], see also the references therein. In fact, the relative flow rates in *all* pores, and, possibly, the types of the flowing fluids in some pores must change. Therefore, a capillary pressure change implies myriads of single-pore rearrangements. The time elapsed to pass from one steady-state configuration to another can be substantial. Characteristic time of such a rearrangement of the flow networks *and* consequent modification of their flow properties is called the *redistribution time*.

For the processes with slowly varying water saturation S, the characteristic time scale of the process is large in comparison with the redistribution time and the hypothesis of the equilibrium local distribution of phases in the porous medium can be accepted. In such a case, the functions k_{rw}, k_{ro} and J in the relations (1) can be assumed to be the universal functions (4) of the water saturation S only. However for the fast processes, among which are such practically important processes as the capillary imbibition or the forced water-oil displacement near the front, the characteristic time scale of the process is often of the order of the redistribution time, and the classical approach based on the relations (4) becomes invalid.

For such non-equilibrium processes, the actual values of the relative phase permeabilities and the Leverett function

defining the flux at a given water saturation can be significantly different from the respective values of the functions k_{rw}, k_{ro} and J obtained by steady-state measurements or by some other experiment based on the assumption of the universality of relationships (4).

There are, however, some instructive properties of the functions $k_{rw}(S)$, $k_{ro}(S)$ and $J(S)$ determined under equilibrium conditions which allow one to avoid this difficulty. Namely, the structure of the function $k_{rw}(S)$ allows one to claim that *for every non-equilibrium process there exists an effective saturation η, generally speaking different from S, and such that the non-equilibrium value of k_{rw} is equal to the equilibrium value of the same function k_{rw} evaluated, however, not for the actual saturation S but for this effective saturation η*. In fact, according to the basic assumptions, the oil, being a component of the effective porous medium, creates an additional drag. However, the function k_{rw} is a monotone function of water saturation, which varies from zero to one. Therefore, there exists a certain value η, for which $k_{rw}(\eta)$ is equal to the non-equilibrium relative permeability of water. For the functions k_{ro} and J, analogous statements are valid as well. An important point here is that the effective saturation η is always higher than the actual saturation S, or equal to it (Figure 2) for all three quantities.
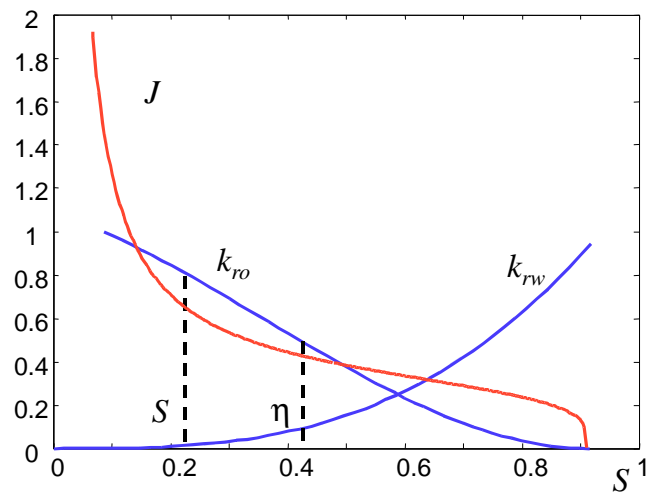
Fig. 2. The effective saturation η is always higher than the actual saturation S.

Indeed, we consider here only immiscible two-phase flow in a porous medium, like water-oil displacement, where the local water saturation is increasing, or at least non-decreasing, so that $\dfrac{\partial S}{\partial t} \geq 0$. For the forced water-oil displacement, a certain part of the water temporarily occupies the wider channels, which in the future will be retaken by the oil. Therefore, the relative water permeability for a non-equilibrium process should be higher, or at least not less than for the equilibrium one. At the same time, the oil temporarily

occupies the narrow channels, which it will release in favor of the water after the redistribution of the fluids. Therefore, the relative oil permeability should be less, or at least not larger than the equilibrium one. Due to the same reasons, the non-equilibrium capillary pressure (Leverett function) is less (not higher at least) than the equilibrium one.

In spontaneous imbibition, a *genuinely non-equilibrium process,* the capillary pressure decreases in *very small* decrements as quickly as the flow networks can adapt. As a result, the water saturation increases monotonically, the hydraulic conductance of the water flow network also increases monotonically, while the reverse is true for the oil flow networks (there can be many of them, each connected to the inlet, the outlet, or both). It just takes some time for these changes to occur. Therefore we may introduce an *effective* water saturation η, and evaluate all the macroscopic flow functions, i.e., both relative permeabilities and the capillary pressure at that saturation. As a result, in a non-equilibrium flow, the water relative permeability will always be higher, the oil relative permeability will always be lower, and the macroscopic driving force for the imbibition, the capillary pressure, will also be lower.

This positive shift from the actual saturation to the effective one could be, generally speaking, different for all three functions k_{rw}, k_{ro} and J. We will make however an essential simplifying assumption: *for all three functions k_{rw}, k_{ro} and J the effective saturations are identical, so* that instead of the relationships (4) we have

$$k_{rw} = k_{rw}(\eta) \quad k_{ro} = k_{ro}(\eta) \qquad J = J(\eta) \qquad (5)$$

where the functions k_{rw}, k_{ro} and J are the universal functions measured under the conditions of the steady-state flow.

The new model will be closed if we provide a relationship between the actual saturation S and the effective saturation η. The difference $\eta - S$ is, in fact, a complicated functional reflecting the whole history of the process. Bearing in mind the first order non-equilibrium effects, we will assume that difference $\eta - S$ depends on the current saturation S, the current saturation rate $\dfrac{\partial S}{\partial t}$ and the redistribution time τ. Thus, we get

$$\eta - S = \mathrm{X}\left(S, \tau, \frac{\partial S}{\partial t}\right) \qquad (6)$$

where X is a dimensionless function. From dimensional analysis, we obtain

$$\eta - S = \Psi\left(S, \tau \frac{\partial S}{\partial t}\right) \qquad (7)$$

where Ψ is another dimensionless function. Obviously, $\Psi(S,0) = 0$, because $\eta = S$ for the processes with a steady

saturation. Furthermore, the argument $\tau \dfrac{\partial S}{\partial t}$ can be represented in the form $\dfrac{\tau}{T}\dfrac{\partial S}{\partial \vartheta}$, where T is the characteristic time scale of the process, and $\vartheta = \dfrac{t}{T}$ is the dimensionless time based on this global time scale. For the non-equilibrium processes slightly deviating from equilibrium, we have the basic dimensional parameter

$$\varepsilon = \frac{\tau}{T} \qquad (8)$$

much less than unity: $\varepsilon \ll 1$. Expanding (7) in the small parameter ε and restricting ourselves by the first term of the expansion, we obtain

$$\eta - S = g(S)\tau\frac{\partial S}{\partial t} \qquad (9)$$

where $g(S)$ is a certain function of the actual water saturation. We can assume further that $g(S)=1$, because the redistribution time τ is a quantity defined only with the accuracy of a factor of the order of unity, and, moreover, generally speaking the redistribution time is also saturation-dependent (see below). Hence, we come to the final relation, which will be used in the whole further presentation

$$\eta - S = \tau\frac{\partial S}{\partial t} \qquad (10)$$

Moreover, for simplicity, at first we will assume that $\tau = const$. In fact, at endpoint saturations $S=S_*$ and $S=S^*$ the redistribution time can have a singular behavior, and some interesting effects are related to this behavior, as we will see below.

Thus, the system (1), (2), (5) and (10) is closed, and it will be considered as a basic mathematical model for our further consideration of the non-equilibrium two-phase flows in porous media. The system can be reduced to the form of equations (3), (10) complemented by the equation

$$\frac{\partial S}{\partial t} + \frac{1}{\phi}\nabla \cdot [uF(\eta)] = a^2\nabla^2\Phi(\eta) \qquad (11)$$

where $a^2 = \dfrac{\gamma}{\mu_w}\sqrt{\dfrac{k}{\phi}}$ and the functions F and Φ are defined by the following relationships:

$$F(S) = \frac{k_{ro}(S)}{k_{ro}(S) + \mu k_{rw}(S)} \qquad (12)$$

$$\Phi(S) = -\int_0^S F(\zeta) k_{ro}(\zeta) J'(\zeta) d\zeta \qquad (13)$$

In equation (12), $\mu = \dfrac{\mu_o}{\mu_w}$. The functions (12)-(13) are also plotted in Fig. 1. Both curves F and Φ have a characteristic S-shaped form and multiple zero derivatives at $S=S_*$ and $S=S^*$. Furthermore, $F(S^*)=1$.

Note that from Eq. (10) we obtain that in the first approximation

$$\eta = S(t + \tau) \qquad (14)$$

so that according to the accepted mathematical model the effective saturation η *is the saturation at a certain time ahead, not behind the actual time*. There is no paradox here: the redistribution of the channels between the fluids described above is a more complicated process than the memory of the saturation evolution.

From equations (11) and (10), the basic equation for the effective water saturation η can be obtained:

$$\frac{\partial \eta}{\partial t} + \frac{1}{\phi} \nabla \cdot \left[u \left(F(\eta) + \tau \frac{\partial}{\partial t} F(\eta) \right) \right]$$
$$= a^2 \nabla^2 \left[\Phi(\eta) + \tau \frac{\partial}{\partial t} \Phi(\eta) \right] \qquad (15)$$

There is, however, a delicate point. Equation (15) is an evolution equation, and for solving it an initial condition is needed

$$\eta(0, x) = \eta_0(x) \qquad (16)$$

However, it is *impossible to prescribe this initial condition, because the effective saturation is not, in fact, a measurable quantity*. What can be considered as a quantity, which can be prescribed is the actual water saturation only because the latter is directly measurable:

$$S(0, x) = S_0(x) \qquad (17)$$

However, using Eq. (17) we can obtain the initial distribution of the effective saturation (16). Indeed, relationship (10) implies $\dfrac{\partial S}{\partial t} = \dfrac{\eta - S}{\tau}$. Putting this into equation (11) at $t=0$ we obtain an elliptic differential equation:

$$\eta_0 + \frac{\tau}{\phi} \nabla \cdot \left[u F(\eta_0) \right] - \tau a^2 \nabla^2 \Phi(\eta_0) = S_0(x) \qquad (18)$$

By solving this equation with respect to η_0, the initial distribution of the effective saturation $\eta_0(x)$ can be obtained. We remind that $S_0(x)$ on the right-hand side of equation (18) is

a known function. It is essential that in equation (18) the actual water saturation $S_0(x)$ can be, in general, discontinuous at the boundary $\partial\Omega$, whereas the function $\eta_0(x)$ is continuous.

The physical model and the basic equations presented here are a modified version of the model presented in papers [5] and [6], see also Ref. [3]. An important experimental verification of this model was performed in the instructive work by Bocharov, Vitovsky and Kuznetsov [7]. Numerical computations of non-equilibrium flows based on this model were performed by Bocharov, Kuznetsov, Chekhovich [8]. The character of the modification and the modified problem statements will be clear after the consideration of the most important special cases: the countercurrent capillary imbibition of a porous block filled initially by oil, and the flow near the water-oil displacement front

2.4 Capillary countercurrent imbibition. For this flow, due to the incompressibility of both fluids, the bulk fluid flux is identically equal to zero:

$$u = u_w + u_o \equiv 0 \qquad (19)$$

Therefore, the basic equation for the effective water saturation (15) reduces to the form

$$\phi \frac{\partial \eta}{\partial t} = a^2 \nabla^2 \left[\Phi(\eta) + \tau \frac{\partial}{\partial t} \Phi(\eta) \right] \qquad (20)$$

and the initial condition is obtained by solving the equation

$$\eta_0(x) - \tau a^2 \nabla^2 \Phi(\eta_0(x)) = S_0(x) \qquad (21)$$

under the appropriate boundary conditions. This problem was considered in [9], and equation (20) was obtained and qualitatively investigated there. A rigorous mathematical investigation was performed in [10] and in [11].

We will illustrate now the basic qualitative differences between classic equilibrium model ($\tau = 0$), and the proposed non-equilibrium model ($\tau > 0$). According to the classical model, the water saturation S satisfies the nonlinear parabolic equation

$$\frac{\partial S}{\partial t} = a^2 \nabla^2 \Phi(S) \qquad (22)$$

This equation was obtained by Ryzhik [12]. Due to the properties of function $\Phi(S)$ formulated above, for an initial condition identically equal to zero outside a certain finite domain, the solution to the equation (22) also vanishes outside a certain finite domain, depending on time. In particular, if we consider a three-dimensional porous block Ω, which initially does not contain water, so that $S_0(x) \equiv 0$ in Ω, and at $t=0$ the boundaries become open for the water, the latter will propagate into the block gradually. It means that at small t, the region near the boundary where the water is penetrated is arbitrarily narrow.

It is not so when the redistribution time τ is positive and constant. As shown in [9-11], the initial *effective* saturation is different from zero in a finite, not arbitrarily narrow region near the boundary of the domain Ω at arbitrarily small $t>0$. Therefore, at arbitrarily small times the water penetrates a finite part of the block. In particular, if the size of the block is not large, the water can penetrate the whole block instantaneously. If the block is large enough and cannot be imbibed instantaneously, then the property of the finite speed of water propagation is preserved.

Consider now the special case of one-dimensional non-equilibrium countercurrent water imbibition to a large initially oil-saturated porous block contacting water at the inlet face. Then, equation (11) takes the following from:

$$\frac{\partial S}{\partial t} = a^2 \frac{\partial^2 \Phi(\eta)}{\partial x^2}, \qquad (23)$$

Here the x axis is directed into the block orthogonally to its inlet face. This problem was considered in [9] under the assumption that the redistribution time τ is a positive constant and the equation for the initial distribution of the effective saturation had the form

$$\tau a^2 \frac{d^2 \Phi(\eta_0(x))}{dx^2} - \eta_0(x) + S_0(x) = 0 \qquad (24)$$

where $S_0(x)$ is the initial distribution of the actual water saturation. Of special interest is the case when the initial water saturation is zero. Then, as shown in [9], the function $\eta_0(x)$ - the initial effective saturation - is different from zero on a certain interval $0 \leq x \leq x_0$, where x_0 is a positive constant obtained in the course of solution (Fig. 3). This means that the water penetrates into a certain part of the block instantaneously, but further it propagates into the block with a finite speed.

However, the assumption that the redistribution time is constant cannot be justified in the case where the initial saturation is zero. At low saturations, water flows through the narrowest flow paths in corners of porous space and the capillary pressure is high. Therefore, the time needed to reconfigure water distribution is small and the relaxation time goes to zero as the saturation of the wetting fluid approaches its minimal value. As the oil saturation is approaching its residual value, the capillary forces weaken and the connectivity of oil grows sparse; hence, the redistribution time increases infinitely. On the major part of the interval between the endpoint saturations, the variation of relaxation time is relatively small. The dependence of relaxation time on water saturation was considered earlier in [7, 8] in the context of numerical simulations of forced water-oil displacement. In both papers, it was assumed that $\tau(0) = 0$.
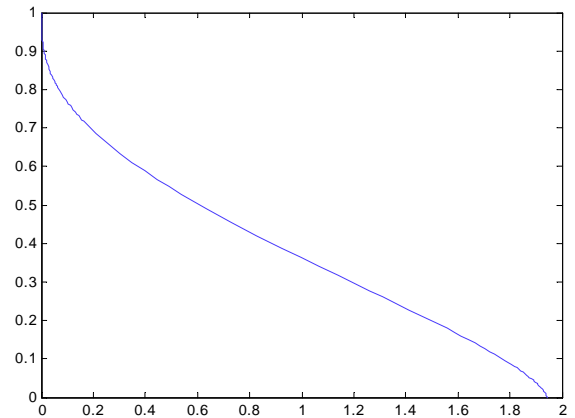

Fig. 3. The effective saturation $\eta_0(x)$ is different from zero in a finite region at arbitrary small time.

By this reason, in [13] the problem of water imbibition into a semi-infinite initially oil-saturated block was reconsidered under the assumption that τ is a function of saturation vanishing at $S=0$ and going to infinity at S close to one. An additional assumption of the power-law saturation dependence at low saturation was accepted. Under these assumptions, the initial value of the effective saturation is also zero according to Eq. (10).

In [13], the asymptotic solutions were obtained at small and large times. It was assumed that for most part of the interval $(0,1)$ the relaxation time is close to a constant value τ_0, whereas at S_* and S^* is goes to zero and infinity, respectively. Therefore, at large t, on a major portion of the interval of water penetration, the actual instantaneous saturation exceeds the threshold, above which the relaxation time is equal to a constant value τ_0. For this case, it was obtained that the cumulative oil recovery through the inlet face of the block can be expressed as

$$R(t) \approx V_0 \left(1 - e^{-\frac{t}{\tau_0}}\right)\sqrt{\frac{t}{\tau_0}} \qquad (25)$$

Here $R(t)$ is the dimensionless ratio of the volume of recovered oil and the initial volume of the oil contained in the block. The coefficient V_0 was expressed through the parameters of the model. It depends on the phase relative permeabilities and viscosities, on the absolute permeability and porosity of the rock, on the inlet face area, on the relaxation time and Leverett's function J. By virtue of equation (25), at $t \rightarrow \infty$ the oil recovery from the block is approximately proportional to the square root of time, i.e. the time scaling suggested by the classical model.

Equation (25) was verified against the results of laboratory measurements reported in [14]. The detailed data were kindly provided to us by the authors. Experiments were performed with samples obtained from near surface diatomite formation

near Lompoc, CA. The samples were cut in a direction parallel to the bedding plane and shaped into cylinders with the diameters of 2.5 cm and the length of 9.5 cm. The porosity of the rock samples was about 70%, the absolute permeability was about 6 mD. For imbibition experiments, the samples were dried and oil was pumped into the pore space. After fully saturating a sample with oil, water was pumped through one endcap while the other endcap was sealed. The oil recovered by countercurrent imbibition was removed by the flowing water.

We matched the measured fractional oil recovery versus time using Eq. (25). Our fitting parameters were the relaxation time τ_0 and the coefficient V_0. In Fig. 4, the data points from different experiments collapse into a single curve based on formula (25). The relaxation time τ_0 was estimated at about 8 s for imbibing a dry sample with no oil, at about 220 s for a sample filled with blandol and at approximately 1230 s for the water-decane pair. For comparison, the same data are plotted in Fig. 5 versus the dimensionless time proposed in [14]:

$$t_D = t \sqrt{\frac{k}{\phi}} \, \frac{\gamma}{L^2} \sqrt{\frac{k_{ro}}{\mu_o} \frac{k_{rw}}{\mu_w}} \, \frac{1}{\sqrt{\frac{k_{ro}}{k_{rw}} \frac{\mu_w}{\mu_o}} + \sqrt{\frac{k_{rw}}{k_{ro}} \frac{\mu_o}{\mu_w}}} \qquad (26)$$

Here L is a characteristic length scale of the sample. For scaling, some characteristic values of the parameters depending on water saturation were used in [14].
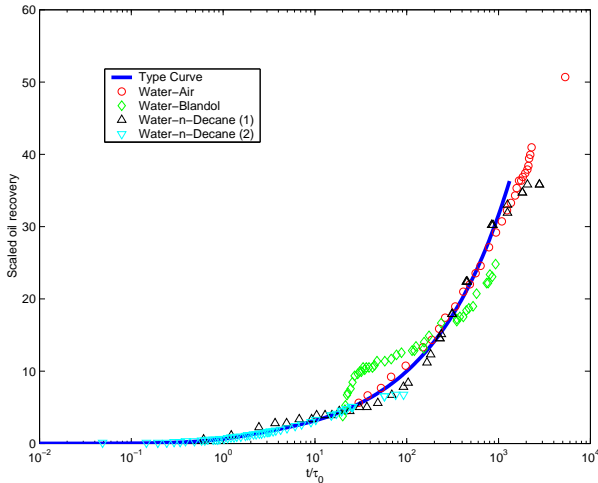

Fig. 4. Matching scaled oil recovery reported by Zhou et al [14]. The type curve corresponds to Eq. (25).
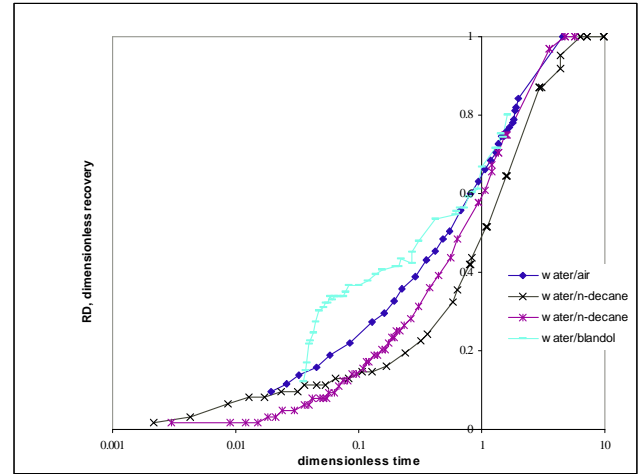

Fig. 5. The data curves from Fig. 4 plotted versus the dimensionless time (26) proposed in the work [14].

Discussion. Laboratory experiments with spontaneous imbibition of various rock samples were broadly reported in the literature. In [15] and [16], the scaling laws developed from the classical model in [17] were used for interpretation of the laboratory coreflood experiments at a reservoir length scale. However, the oil recovery curves in countercurrent imbibition experiments did not follow the classical square root of time rule, especially at early times, see e.g., [18]. This phenomenon was investigated experimentally and confirmed in numerous following works, see [14], [19-24]. As it has been mentioned above, the non-equilibrium effects reduce if the viscosities of the fluids are low and the laboratory sample permeability is high. This is confirmed by experiments with imbibition of dry cement pastes by water [25].

The influence of the temperature conditions and the presence of chemical additives on countercurrent imbibition oil recovery was studied experimentally in [26] and [27].

Within the model overviewed in this paper, the deviation of the oil recovery curve from the classical square root of time rule has been explained by the non-equilibrium nature of the process of imbibition. Attempts to model and numerically simulate this phenomenon based on the classical model also were undertaken by the researchers. Numerical studies of countercurrent imbibition were started in early works [28], [29]. In [30] the numerical simulations were extended to 2D flow. All these papers were based on the classical approach. A comparative survey of other works, also based on the classical model with minor variations, was presented in [31]. In [32], [33], a "delayed" inlet boundary condition was introduced that allowed to match the laboratory data using the classical approach. In these papers, this condition was derived from an empirical relationship proposed in [34]. The same approach made possible good matching of the data in [35]. In fact, this "delayed" boundary condition was derived from the non-equilibrium spontaneous imbibition model in [9] and the exponential decay rate was explicitly linked to the relaxation time.

A system of equations characterizing multiphase flow in porous media was obtained in [36]. This system is incomplete, however, an important observation derived in this work from the pore-scale thermodynamic analysis of consequences of fast-changing saturations was that at transient conditions, the capillary pressure calculated using the curve obtained at steady-state flow has to be modified by a term proportional to saturation rate change. The development of this model was continued in [37-39] and a mathematical investigation was performed in [40]. Formally, the last equation (5) and (10) can be related to the model proposed in these subsequent works, but they do not take into account the changes in relative permeabilities. Also, neither a characteristic redistribution time nor effective saturation are introduced, so the derivations of the model proposed in [36], and the one overviewed in this paper are fundamentally different.

In [22], a drag force caused by simultaneous flow of two fluids in a porous stratum was incorporated into the Darcy's law. As a result, the permeability coefficient became a tensor even for an isotropic rock. Although the numerical simulations based on this assumption matched the data measured in individual countercurrent imbibition experiments, the universality of the cross-terms in the permeability tensor was not established in [22].

2.5 Forced water-oil displacement.

The mathematical model of the forced water-oil displacement taking into account non-equilibrium effects was presented in [6], a complete rigorous mathematical investigation was performed in [41]. An important special case where the direct capillary pressure effects can be neglected (but not in relative permeabilties) was rigorously investigated in [42].

The most important manifestation of the non-equilibrium effects consists in the following.

In the classical paper by Rapoport and Leas [43], the concept of *stabilized zone* around the water-oil displacement front was introduced. The stabilized zone is a quasi-steady region around the displacement front (a region of sharp variation of the water saturation), which determines the structure of the transition between injected water and oil. In the subsequent paper [44], the stabilized zone was obtained as a rigorous solution of the traveling-wave type of the equation of the Muskat-Leverett model.

A paradoxical property of the results obtained in [43] and [44] was that the width Λ of the stabilized zone where the saturation sharply changes appears to be decreasing inversely proportional to the displacement speed v, Fig. 6, a. This result seems to be unnatural. Taking into account the non-equilibrium effects (the redistribution time) led to a different and more natural result, Fig. 6, b. The stabilized zone around the displacement front also is obtained, but its width with growing velocity at first is decreasing, than it reaches a certain minimum, and after that starts to increase. In the limiting case of large displacement speeds, when the capillary pressure become negligible (but the capillary effects in relative permeabilities are still preserved) the width of the stabilized zone is increasing linearly with the displacement speed, until it

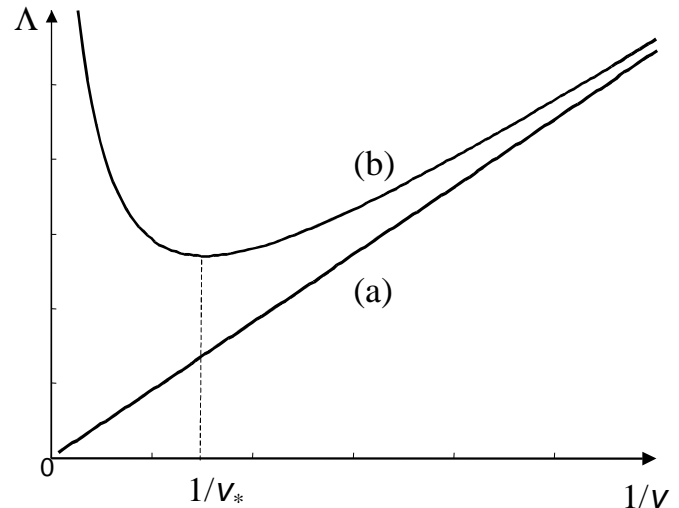becomes comparable with the distance between the wells, and the stabilized zone disappears.

Fig. 6. The width of stabilized zone is proportional to $1/v$ according to Rapoport-Leas model (a), and has a global minimum at a certain $v=v_*$ according to the non-equilibrium model (b).

Conclusions

An overview of the theory of non-equilibrium water-oil (or, more generally, wetting – non-wetting fluid) displacement proposed and developed in [5, 6, 9, 11, 41] has been presented. The non-equilibrium effects become important when the characteristic transition times become comparable or smaller than the time needed for the redistribution of the fluids in flow paths inside the pore space. Two practically very important processes of this type have been considered: spontaneous countercurrent imbibition and flow at the front of the forced water-oil displacement. A rigorous presentation of the respective physical phenomena and their mathematical models has been performed. The results of a comparison with experiments are presented. The effects of dependence of the relaxation time on the water saturation have been discussed and the corresponding modification of the problem formulation is presented.

To summarize, the presented discussion of non-equilibrium effects in water-oil displacement presented in this paper shows that these effects are of primary importance for such processes as countercurrent capillary imbibition and forced water-oil displacement.

Acknowledgements

Nomenclature

a^2 = diffusivity coefficient, L^2/t

F = fractional flow function, dimensionless

g = dimensionless function

J = Leverett's function, dimensionless

k = absolute permeability of the rock, L^2

k_{rw} = relative permeability to oil, dimensionless

k_{rw} = relative permeability to water, dimensionless

L = characteristic length, L

p_o = oil pressure, m/Lt^2

p_w = water pressure, m/Lt^2

$R(t)$ = dimensionless cumulative oil recovery

S = actual water saturation, dimensionless

S_0 = initial water saturation, dimensionless

u = total flux, L/t

u_o = oil flux, L/t

u_w = water flux, L/t

t = time, t

t_D = dimensionless time

T = process characteristic time, t

v = water-oil front velocity, L/t

V_0 = dimensionless volume

ε_N = dimensionless time

γ = oil-water interface surface tension, m/t^2

ϕ = rock porosity, dimensionless

Φ = dimensionless function

Ψ = dimensionless function

X = dimensionless function

Λ = the width of stabilized zone, L

μ = the ratio μ_o / μ_w, dimensionless

μ_o = oil viscosity, m/Lt

μ_w = water viscosity, m/Lt

η = effective water saturation, dimensionless

η_0 = initial effective water saturation, dimensionless

τ = redistribution time, t

Ω = a domain in a porous medium

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